ISSN 2229-5518

MHD flow of Cu-water Nanofluid over a Stretching Sheet with Second Order Slip Condition.

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Abstract: The study of boundary layer flow over a stretching sheet is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. Examples, glass-fiber production, aerodynamics, wire drawing, paper production, plastic sheets, metal sheets and many others.The boundary layer flow over a stretching sheet in presence of a magnetic field has been presented in this chapter under the velocity slip conditions. Similarity transformations are used to convert the partial differential equations corresponding to the momentum and energy equations into non-linear ordinary differential equations whose numerical solutions has been obtained using fourth order Runge-Kutta method with shooting techniques. It has been observed that the velocity decreases with increasing slip parameter as well as with the increasing magnetic parameter. Temperature increases with the increasing values of magnetic parameter.

Index terms: MHD; Nanofluid ; Runge Kutta Method; Stretching Sheet; velocity slip condition.

1. Introduction

The study of laminar flow and heat transfer over a stretching sheet in a viscous fluid is of considerable interest because of its ever increasing industrial applications and important bearings on several technological processes. such as paper production, glass industries, plastic films and polymer extrusion. In this regard Crane (1) investigated the flow caused by the stretching of a sheet. Chen and Char (2), extended the work of Crane by including the effect of heat and mass transfer analysis under different physical situations.. Magyari and Keller (3) focused on heat and mass transfer on boundary layer flow due to an exponentially continuous stretching sheet. Elbashbeshy (4) studied the flow past an exponentially stretching surface. Vajravelu and Cannon (5) investigated the fluid flow over a nonlinear stretching sheet. Bidin and Nazar (6) studied the effect of thermal radiation on the steady laminar two-dimensional boundary layer flow and heat transfer over an exponentially stretching sheet.

All the above mentioned studies continued their discussions by assuming the no slip boundary conditions. But it has been found that slip condition is a common phenomenon and it occurs almost in every flow therefore in our work we have stressed on slip condition. It has also been observed that many studies are available on slip condition of the flow. Sahoo and Poncet (7) addressed the flow of third grade fluid past an exponentially stretching sheet with partial slip boundary condition. Recently, Mukhopadhyay and Gorla (8) analyzed the effects of partial slip on flow past an exponentially stretching sheet. Sharma and Ishak, (9) studied the second order slip flow of Cu-Water nanofluid over a stretching sheet with heat transfer,

It appears that no attempt has been made to analyze the effects of second order slip condition on MHD flow of Cu-water nanofluid over a stretching sheet so it is considered in this paper. Using similarity transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. Using shooting method numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. The analysis of the results obtained shows that the flow field is influenced appreciably by the slip parameter in the presence of magnetic field. Estimation of skin friction which is very important from the industrial application point of view is also presented in this analysis. It is hoped that the results obtained will not only provide useful information for applications, but also serve as a complement to the previous studies.

ISSN 2229-5518

2. Formulation and Solution of the Problem

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We consider a steady two-dimensional laminar boundary layer flow of nanofluids over a stretching sheet with a linear velocity $u_w(x) = cx$, where $c > 0$ is a constant and x is the coordinate measured along the stretching surface and the flow takes place at $y > 0$, where y is the coordinate measured normal to the stretching rate. Two equal and opposite forces are applied along the x-axis so that the sheet is stretched keeping the origin fixed. The fluid is electrically conducting under the influence of an applied magnetic field $B_0(x)$ normal to the stretching surface. It is assumed that the base fluid and the nanoparticles are in thermal equilibrium. The thermophysical properties of the base fluid water and nanoparticle of Cu are given in Table 1.

Table 1: Thermophysical properties of base fluid water and Cu nano particle.

Physical properties	Water (H_2O)	Copper (Cu)
$\rho(kg/m^3)$	997.1	8933
$C_\rho(J/kg\ \kappa)$	4179	386
$K(w/m\kappa)$	0.613	400

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Under the usual boundary layer approximations, the continuity, momentum and energy equations for nanofluids in Cartesian coordinates x and y can be written as:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2}-\sigma\frac{B_0^2(x)}{\rho_{nf}}u \quad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\frac{\partial^2 T}{\partial y^2} \quad (3)$$

Subjected to the boundary conditions:

$u = u_w + U_{slip}$, $v=0$, $T = T_w = T_\infty + c\left(\frac{x}{l}\right)^2$ at $y = 0$

$$U_{slip} = \frac{2}{3}\left(\frac{3-\alpha l^3}{\alpha} - \frac{3}{2}\left(\frac{1-l^2}{k_n}\right)\right)\lambda\frac{\partial u}{\partial y} - \frac{1}{4}\left(l^4 + 2\left(\frac{1-l^2}{k_n^{\,2}}\right)\right)\lambda^2\frac{\partial u}{\partial y}$$

$$= A\frac{\partial u}{\partial y} + B\frac{\partial^2 u}{\partial y^2} \qquad (4)$$

where ,

u and v are the velocity components in the x and y directions respectively.

T = Temperature of the nanofluid

T_∞ = Temperature of the nanofluid far away from the sheet.

B_0 = The uniform magnetic field strength.

σ = Electrical conductivity of base fluid.

μ_{nf} = Dynamic viscosity of the nanofluid.

ρ_{nf} = Density of the nanofluid.

α_{nf} = Thermal diffusivity of the nanofluid.

U_{slip}= Second order velocity slip.

For the nanofluids K_{nf} is the thermal conductivity and $(\rho C_p)_{nf}$ is the heat capacity, which are given by

$$\alpha_{nf} = \frac{K_{nf}}{(\rho C_\rho)_{nf}} \qquad (5)$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2..5}} \qquad (6)$$

The effective density of the nanofluid is given by

$$(\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi\,(\rho C_p)_s \qquad (7)$$

$$\frac{k_{nf}}{k_f} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \qquad (8)$$

3. Solution of the Problem:

To simplify the mathematical analysis of our study we introduce the following similarity transformations

$$\psi(x, y) = f(\eta)\, x\,(v_f c)^{1/2} \qquad u = c\,x\,f'(\eta)$$

$$v = -f(\eta)(v_f c)^{1/2} \qquad \eta = y\,(c/v_f)^{1/2}$$

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$$\theta(\eta) = \frac{T - T_\infty}{T - T_w} \qquad (9)$$

$$u = \frac{\partial \psi}{\partial y},\ v = -\frac{\partial \psi}{\partial x}$$

Where $\psi(x, y)$ is the stream function with

$\theta(\eta)$ = dimensionless temperature.

$f(\eta)$ = dimensionless velocity

Where primes denote differentiation with respect to the similarity variable η.

Making use of equation (9) ,the continuity equation is automatically satisfied and equations (2) ,(3) under boundary conditions (4) becomes

$$f''' + ff'' - f'^2 - Mf' = 0 \qquad (10)$$

Where, $M = \sigma \frac{B_0^2(x)}{\rho_{nf} c}$

$$\frac{k_{nf}}{k_f}\theta'' + \Pr\left((1-\phi) + \phi\frac{(\rho c_p)_s}{(\rho c_p)_f}\right)(f\theta' - 2f'\theta) = 0 \qquad (11).$$

Subject to the following boundary conditions

$$f(0) = 0$$

$$f'(0) = 1 + \gamma f''(0) + \delta f'''(0) \qquad \theta(0) = 1$$

$$f'(\infty) \to 0 \qquad \theta(\infty) \to 0 \qquad (12)$$

Where ,

$\gamma = A(c/v_f)^{1/2} > 0$ is the first order velocity slip parameter

$\delta = B(c/v_f) < 0$ is the second order velocity slip parameter

$\Pr = \frac{\mu_f}{k_f}$ is Prandtl number.

The physical quantities of interest in this problem are the local skin friction coefficient Cf and the Nusselt number Nux, which are defined as

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$$C_f = \frac{\tau_w}{\rho_f u_w^2} \qquad Nu = \frac{q_w}{k_f (T_w - T_\infty)} \tag{13}$$

Where ,

τ_w is the surface shear stress and q_w is the surface heat flux ,which are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0} \qquad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0} \tag{14}$$

Using the similarity variable (9) ,we obtain

$$\mathrm{Re}_x^{1/2} C_f = \frac{1}{(1-\phi)^{2.5}} f''(0) \qquad \mathrm{Re}_x^{-1/2} Nu = -\frac{k_{nf}}{k_f}\theta'(0) \tag{15}$$

$\mathrm{Re}_x = u_w x / \nu_f$ is the local Reynolds number.

The non linear boundary value problem represented by equations (10) and (11) is solved numerically using the fourth order Runge–Kutta method with shooting technique. The numerical solutions are obtained for several values of the governing parameters, like nanoparticle solid volume fraction parameter, magnetic field parameter, velocity slip parameter. In the present work, the default values have been taken as:Pr= 6.2 (water), ϕ = 0.1 ,ϒ=0.5 and δ =-0.5.

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4. RESULT AND DISCUSSION:

In the present work a systematic study of parameters which influence the MHD flow of Cu-water nanofluid over a stretching sheet with second order slip condition such as magnetic parameter nanoparticle concentration, temperature, first and second order slip parameters, has been carried out and the results are shown in Figs. 1-7.

Fig.(1): Shows the influence of magnetic field parameter M on the velocity profile f′(η).As the value of magnetic parameter M increases ,the retarding force increases and consequently the velocity decreases.

Fig.(2): Displays the effect of volume fraction on the dimensionless temperature profile of copper water nanofluid under the influence of magnetic field . Here, it is noted that as volume fraction increases magnitude of dimensionless temperature also increases i,e the volume fraction is directly proportional to the dimensionless temperature .

Fig.(3): Exhibits the effect of Shear stress distribution for various value of M when γ=0.5 ,here it is observed that the magnitude of the wall of shear stress given by **$(1/(1-\Phi)^{2.5})f''(0)$** decreases when the value of magnetic parameter M increases.

Fig.(4): It illustrates the effect of Velocity distribution for various value of magnetic parameter M when first order slip parameter ϒ= 0.5. It is clearly depicts from the fig that velocity reduces with increase in magnetic parameter value.

.**Fig.(5):** Here the effect of first order slip parameter γ on velocity distribution under the influence of magnetic field studied and it is observed that velocity decreases with increasing value of first order slip condition.

Fig.(6): Displays the effect of second oder slip parameter δ on the velocity distribution under the influence of magnetic parameter .It reveals that increasing value of $|\delta|$ decreases the magnitude of velocity.

Fig.(7): I t may be observed from the figure that the magnitude of the wall shear stress $(1/(1-\Phi)^{2.5})f''(0)$ decreases with increasing value of $|\delta|$ for the fixed value of magnetic parameter.

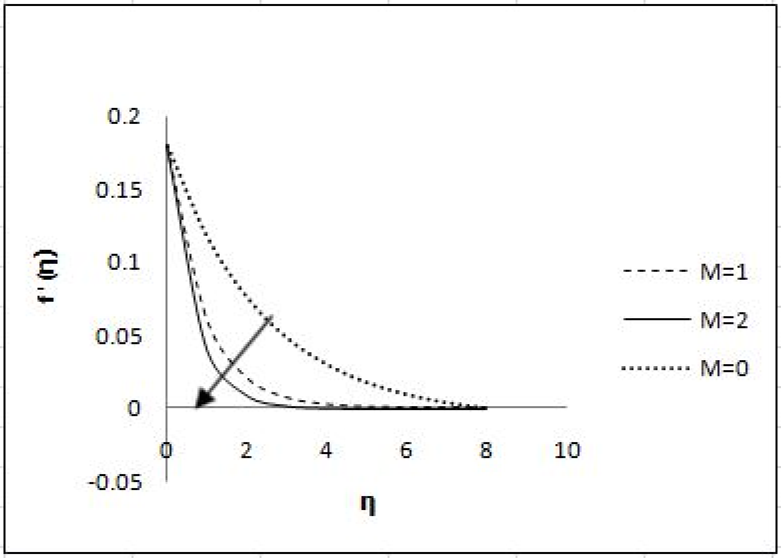

Fig.1 Velocity profile f'(η) for various value of Magnetic parameter

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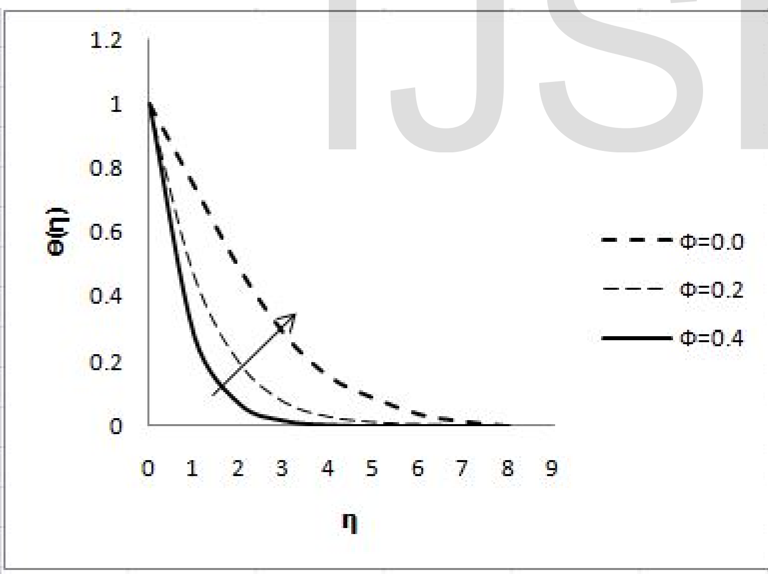

Fig. 2 Temperature profile for various value of volume fraction Φ

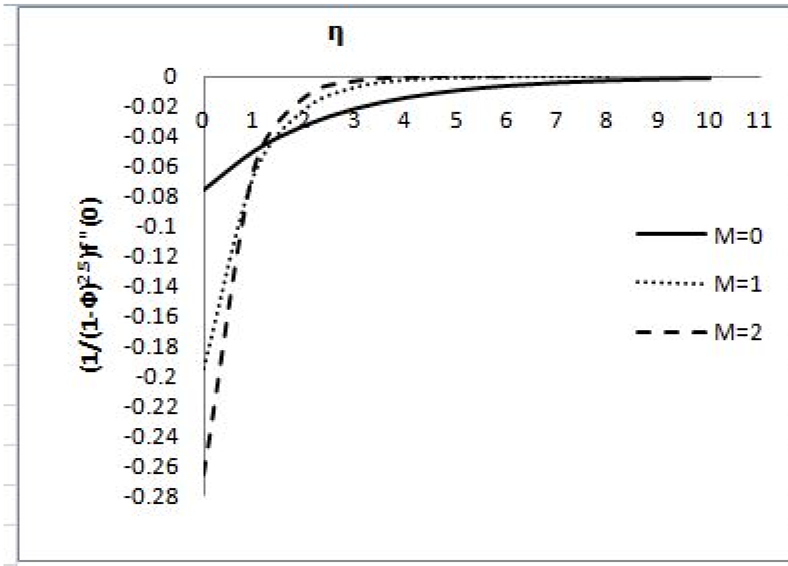

Fig.3 Shear stress distribution for various value of M when γ=0.5

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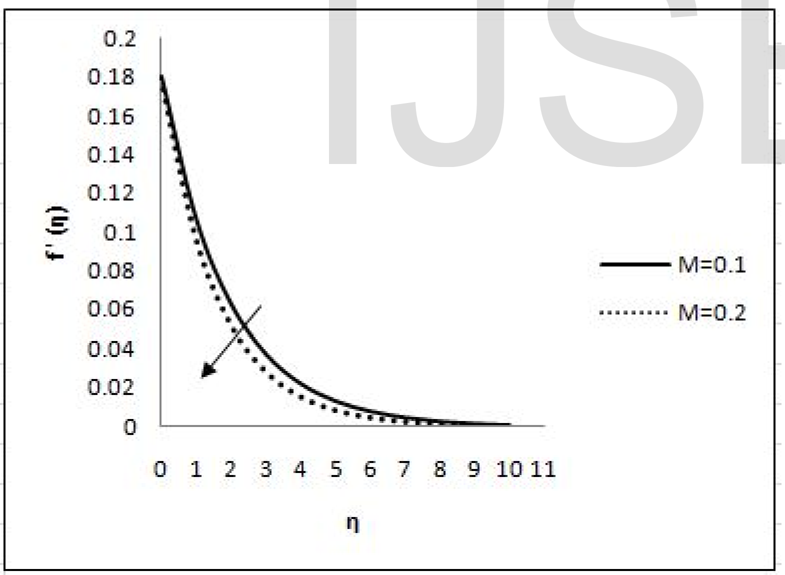

Fig. 4 Velocity distribution for various value of magnetic parameter M when ϒ= 0.5

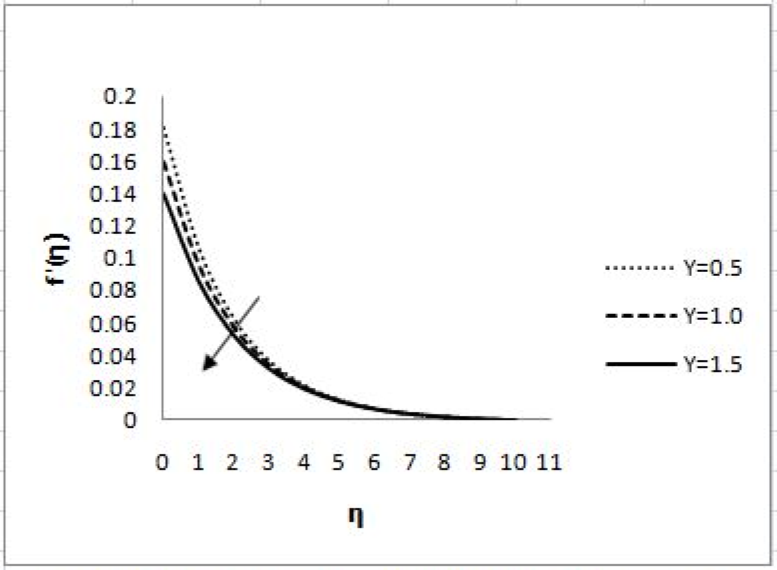

Fig. 5 Velocity distribution for various value of γ under the influence of magnetic field.

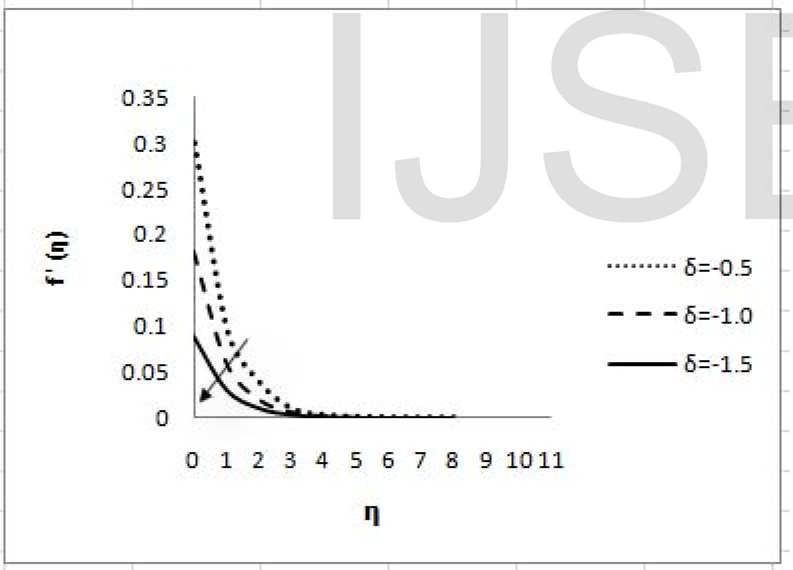

Fig. 6 Velocity distribution for various value of δ when M=1

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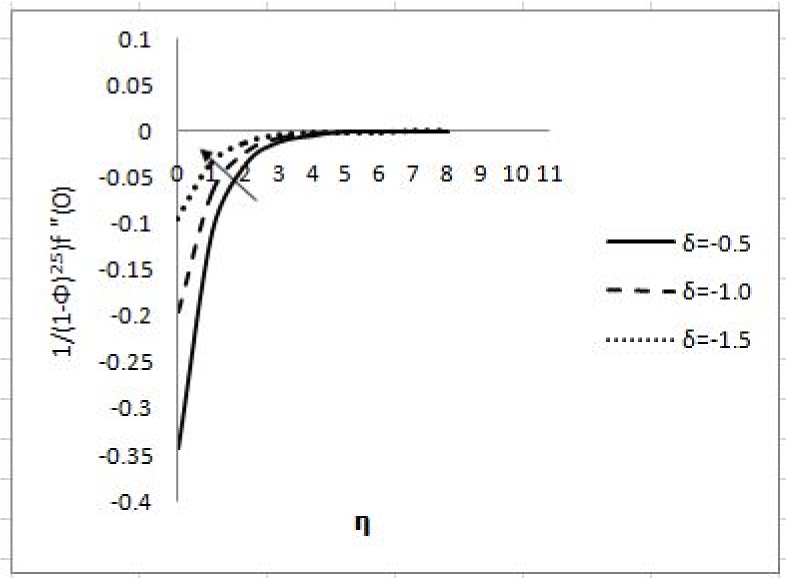

Fig. 7 Shear stress distribution for various value of δ when M=1

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5. CONCLUSIONS

In the present work, the MHD boundary layer flow of Cu-water nanofluid over a stretching sheet with second order slip condition has been analyzed. The governing non linear partial differential equations are transformed to ordinary differential equations using similarity transformation and solved numerically using Runge-Kutta method with shooting technique.The results show that any increase of volume fraction of nanoparticle under the influence of magnetic parameter increased the dimensionless temperature. Dimensionless velocity decreases with increasing value of magnetic field. It is also found that first order slip parameter and second order slip parameter reduces the magnitude of skin friction under the influence of magnetic field.

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